ELSEVIER

International Journal of Solids and Structures 41 (2004) 5647–5661

INTERNATIONAL JOURNAL OF SOLIDS and STRUCTURES

www.elsevier.com/locate/ijsolstr

Buckling and vibration of a plate on elastic foundation subjected to in-plane compression and moving loads

Seong-Min Kim *

Center for Transportation Research, The University of Texas at Austin, 3208 Red River, Suite 200, Austin, TX 78705, USA

Received 11 November 2003; received in revised form 27 April 2004
Available online 11 June 2004

Abstract

The stability and dynamic displacement response of an infinite thin plate resting on a Winkler-type or a two-parameter elastic foundation have been investigated when the system is subjected to in-plane static compressive forces and a distributed load moving with a constant advance velocity. The amplitude of the moving load was assumed to be either constant or of harmonic variation and damping of a linear hysteretic nature for the foundation were considered. Formulations in the transformed field domains of time, space, and moving space were employed. The steady-state response to a moving harmonic load and the response to a moving load of constant amplitude were obtained using a double Fourier transform. Analyses were performed: (i) to investigate the effects of various parameters, such as the load velocity, load frequency, and damping, on the deflected shapes, maximum displacements, and critical values of the velocity, frequency, and in-plane compression, and (ii) to examine how the in-plane compression affects the stability and vibration of the system. Expressions to predict the critical (resonance) velocity, critical frequency, and in-plane buckling force were proposed.

Keywords: Buckling; Critical frequency; Critical velocity; Elastic foundation; Harmonic load; In-plane compression; Moving load; Plate; Transformed field domain

1. Introduction

Many studies have been performed to predict the deflections and stresses of highway and airfield pavement systems by employing a plate on an elastic foundation. The surface layer of the pavement (concrete slab in the rigid pavement and asphalt mixtures in the flexible pavement) is normally modeled based on the Kirchhoff thin plate theory and the underlying layers are modeled using an elastic foundation, normally a Winkler-type elastic foundation. When the shear stiffness of the foundation is considered, the two-parameter foundations, such as Filonenko-Borodich (Filonenko-Borodich, 1940), Pasternak (Pasternak, 1954), generalized (Kerr, 1964), and Vlasov (Vlasov and Leontev, 1966) foundations, can be used. The

* Tel.: +1-512-232-3134; fax: +1-512-232-3151.
E-mail address: seong-min@mail.utexas.edu (S.-M. Kim).

0020-7683/$ - see front matter
doi:10.1016/j.ijsolstr.2004.05.006

loads imposed by vehicles have often been considered as static loads in most studies (Westergaard, 1925; Huang, 1993). Recently, studies have been performed to predict the pavement response to moving loads by employing a plate on an elastic foundation since the pavement systems are subjected to moving loads (Kim and Roesset, 1998; Liu et al., 2000; Kim et al., 2002; Huang and Thambiratnam, 2002). When the moving loads are considered, the load amplitude is often assumed to be constant. However, the moving loads created by vehicles in fact have variations in load amplitude with time that result from the pavement surface roughness and the mechanical systems of the vehicles (Nasim et al., 1991; Kim et al., 2002; Kim and McCullough, 2003). In addition, nondestructive testing vehicles such as the rolling dynamic deflectometer apply a steady-state harmonic force while continuously moving (Bay et al., 1995; Kim et al., 1999). The moving loads with such variations in load amplitude need to be considered in addition to the moving loads of constant amplitude.

When a plate on an elastic foundation is analyzed with moving loads, it is generally assumed that the system is subjected to only moving loads and the forces in the plate's in-plane direction are often ignored. However, most Portland cement concrete pavement systems are subjected to in-plane compressive forces due to temperature and moisture changes. If temperature increases, concrete slabs expand and compressive forces are induced because the slabs push each other. Due to these in-plane compressive forces, the concrete pavements sometimes experience buckling, which is also called blowup (Kerr and Dallis, 1985). Blowups are observed in jointed concrete pavements and continuously reinforced concrete pavements. The in-plane compressive forces are also induced in prestressed concrete pavements due to prestressing (Brunner, 1975; Cable et al., 1986; Powers and Zaniewski, 1987; Okamoto and Tayabji, 1995). Failures of the prestressed concrete pavements caused by high compressive forces sometimes occur. Therefore, the effect of the in-plane compression in the pavement system needs to be investigated when the system is subjected to moving loads.

The main objective of this paper is to discuss the stability and vibration of a plate resting on an elastic foundation subjected to in-plane compressive forces under a moving load with either constant or harmonic amplitude variations. The plate was assumed to extend to infinity in the horizontal plane, and the geometry and material properties were assumed to be linearly elastic. The elastic foundation was considered as either a Winkler-type or a two-parameter foundation and damping of a linear hysteretic nature was considered for the foundation. A distributed load with a constant advance velocity was considered instead of a point load because moving loads in practice have normally a finite area over which they are distributed and the point load represents only an extreme case. Formulations in the transformed field domains were developed and the solutions were obtained using a triple Fourier transform in time, space, and moving space for moving loads with arbitrary load variation, and a double Fourier transform in space and moving space for the steady-state response to moving harmonic loads and for the response to moving loads of constant amplitude. Analyses were performed: (i) to investigate the effects of various parameters, such as the load velocity, load frequency, and damping, on the displacements and critical values of the velocity, frequency, and in-plane compressive force, and (ii) to examine how the in-plane compressive force affects the displacement response. By conducting a large number of parametric studies, equations to predict the critical (resonance) values of the velocity and frequency and the buckling force were developed.

2. Formulations

If a Kirchhoff plate on a Winkler-type elastic foundation is subjected to in-plane static compressive forces and a dynamic load, as shown in Fig. 1, the governing differential equation of the system without damping for the vertical displacements w at time t can be written in a Cartesian coordinate system $\{x, y, z\}$ as

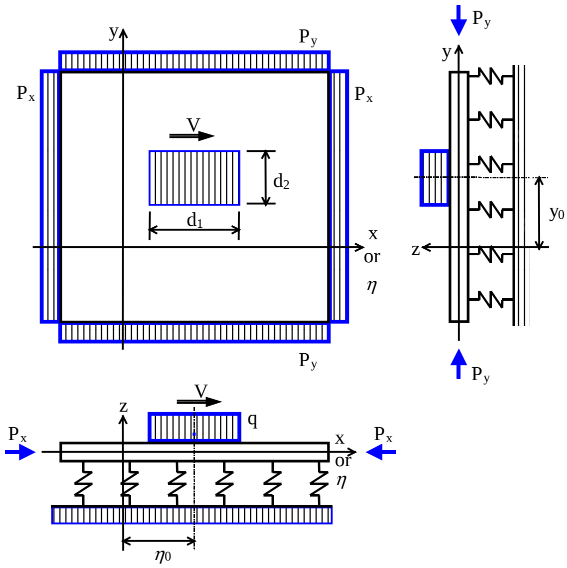

Fig. 1. Plate on elastic foundation subjected to in-plane compression and moving load.

$$D_P\left(\frac{\partial^4 w(x,y,t)}{\partial x^4}+2\frac{\partial^4 w(x,y,t)}{\partial x^2\partial y^2}+\frac{\partial^4 w(x,y,t)}{\partial y^4}\right)+\left(P_x\frac{\partial^2 w(x,y,t)}{\partial x^2}+P_y\frac{\partial^2 w(x,y,t)}{\partial y^2}\right)$$
$$+m\frac{\partial^2 w(x,y,t)}{\partial t^2}+kw(x,y,t)=q(x,y,t) \tag{1}$$

where m is the mass of the plate per unit area, k is the vertical stiffness of the foundation per unit area, q is the vertical load per unit area, P_x and P_y are the in-plane compressive forces in the x- and y-directions per unit length, respectively, and D_P is the flexural rigidity of the plate defined by

$$D_P=\frac{Eh^3}{12(1-v^2)} \tag{2}$$

where E, h and v are the elastic modulus, the thickness and Poisson's ratio of the plate, respectively.

If the loads are moving in the positive x-direction with a constant advance velocity V, a moving coordinate η can be defined by $x-Vt$. Then, the governing differential equation in a moving Cartesian coordinate system $\{\eta,y,z\}$ can be rewritten as

$$D_P\left(\frac{\partial^4 w(\eta,y,t)}{\partial \eta^4}+2\frac{\partial^4 w(\eta,y,t)}{\partial \eta^2\partial y^2}+\frac{\partial^4 w(\eta,y,t)}{\partial y^4}\right)+\left(P_x\frac{\partial^2 w(\eta,y,t)}{\partial \eta^2}+P_y\frac{\partial^2 w(\eta,y,t)}{\partial y^2}\right)$$
$$+m\left(\frac{\partial^2 w(\eta,y,t)}{\partial t^2}-2V\frac{\partial^2 w(\eta,y,t)}{\partial t\partial \eta}+V^2\frac{\partial^2 w(\eta,y,t)}{\partial \eta^2}\right)+kw(\eta,y,t)$$
$$=q(\eta,y,t) \tag{3}$$

The solution of Eq. (3) can be obtained using the Fourier transform if the plate is assumed to extend to infinity in the horizontal plane. If ξ, ζ and Ω are assumed to be the transformed fields of η (moving space), y (fixed space), and t (time), and if $w(\eta,y,t)$ and $q(\eta,y,t)$ are written in the form of $W(\xi,\zeta,\Omega)e^{i\Omega t}e^{i\xi\eta}e^{i\zeta y}$ and $Q(\xi,\zeta,\Omega)e^{i\Omega t}e^{i\xi\eta}e^{i\zeta y}$, where $i=\sqrt{-1}$, the transformed displacements $W(\xi,\zeta,\Omega)$ can be obtained by

$$W(\xi,\zeta,\Omega)=\frac{Q(\xi,\zeta,\Omega)}{D_{\mathrm{P}}(\xi^2+\zeta^2)^2-(P_x\xi^2+P_y\zeta^2)+k-m(\Omega-V\xi)^2} \tag{4}$$

where the transformed load $Q(\xi,\zeta,\Omega)$ is obtained using the triple Fourier transform

$$Q(\xi,\zeta,\Omega)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}q(\eta,y,t)\mathrm{e}^{-\mathrm{i}\xi\eta}\mathrm{e}^{-\mathrm{i}\zeta y}\mathrm{e}^{-\mathrm{i}\Omega t}\,\mathrm{d}\eta\,\mathrm{d}y\,\mathrm{d}t \tag{5}$$

Finally, the dynamic displacement response can be obtained using the triple inverse Fourier transform

$$w(\eta,y,t)=\frac{1}{(2\pi)^3}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\frac{Q(\xi,\zeta,\Omega)}{D_{\mathrm{P}}(\xi^2+\zeta^2)^2-(P_x\xi^2+P_y\zeta^2)+k-m(\Omega-V\xi)^2}\times\mathrm{e}^{\mathrm{i}\xi\eta}\mathrm{e}^{\mathrm{i}\zeta y}\mathrm{e}^{\mathrm{i}\Omega t}\,\mathrm{d}\xi\,\mathrm{d}\zeta\,\mathrm{d}\Omega \tag{6}$$

If viscous damping is considered, $c\frac{\partial w(x,y,t)}{\partial t}$ and $c\left(\frac{\partial w(\eta,y,t)}{\partial t}-V\frac{\partial w(\eta,y,t)}{\partial\eta}\right)$ should be added in Eqs. (1) and (3), respectively, where c is the viscous damping constant. It is generally accepted that most of the dissipation of energy in soils takes place through internal friction (hysteretic damping) rather than through viscous behavior. Hysteretic damping produces an energy loss per cycle that is frequency independent. Since the elastic foundation represents soil deposits in a number of practical applications, frequency-independent linear hysteretic damping has been considered in this study for the foundation with an expression $2iDk$ for the damping term, where D is the damping ratio (Foinquinos and Roesset, 1995). Then, the transformed displacements in Eq. (4) can be rewritten including damping terms as

$$W(\xi,\zeta,\Omega)=\frac{Q(\xi,\zeta,\Omega)}{D_{\mathrm{P}}(\xi^2+\zeta^2)^2-(P_x\xi^2+P_y\zeta^2)+k(1+2iD)-m(\Omega-V\xi)^2+ic(\Omega-V\xi)} \tag{7}$$

It is noted that the sign of the linear hysteretic damping term needs to be consistent with that of the viscous damping term. In practice, the above equations are solved using the fast Fourier transform (FFT), which is a discrete transform.

If the moving load has a harmonic variation of the amplitude $\mathrm{e}^{\mathrm{i}\bar{\Omega}t}$ and only the steady-state response is of interest, the displacement response in Eq. (6) with considering linear hysteretic damping can be rewritten as

$$w(\eta,y,t)=\frac{1}{(2\pi)^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\frac{Q(\xi,\zeta,t)}{D_{\mathrm{P}}(\xi^2+\zeta^2)^2-(P_x\xi^2+P_y\zeta^2)+k(1+2iD)-m(\overline{\Omega}-V\xi)^2}\mathrm{e}^{\mathrm{i}\xi\eta}\mathrm{e}^{\mathrm{i}\zeta y}\,\mathrm{d}\xi\,\mathrm{d}\zeta \tag{8}$$

with

$$Q(\xi,\zeta,t)=\mathrm{e}^{\mathrm{i}\bar{\Omega}t}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}q(\eta,y)\mathrm{e}^{-i\xi\eta}\mathrm{e}^{-\mathrm{i}\zeta y}\,\mathrm{d}\eta\,\mathrm{d}y \tag{9}$$

where $\overline{\Omega}$ is the load frequency. If the response to the force $\sin\overline{\Omega}t$ (the imaginary component of $\mathrm{e}^{\mathrm{i}\bar{\Omega}t}$) is considered, the imaginary component of Eq. (8) should be used. If a moving load of constant amplitude ($\overline{\Omega}=0$) is considered, the response can be obtained by inserting 0 into the load frequency $\overline{\Omega}$ in Eqs. (8) and (9). Because η is a point on the moving axis, the above equations represent the response at a moving point with time. The response at a fixed point can simply be obtained by the relation $\eta = x - Vt$, where x is the fixed point.

If a two-parameter foundation such as a Pasternak foundation (Pasternak, 1954) is considered to include the shear stiffness of the foundation, the terms having the in-plane compressive forces P_x and P_y in Eq. (1) can be rewritten as

$$(P_x-k_2)\frac{\partial^2 w(x,y,t)}{\partial x^2}+(P_y-k_2)\frac{\partial^2 w(x,y,t)}{\partial y^2} \tag{10}$$

where k_2 is the second parameter of the two-parameter foundation. As can be seen from Eq. (10), the second parameter of the foundation reduces the in-plane compressive forces. Therefore, the differences between the in-plane compressive forces and the second parameter of the foundation should be used as the combined compressive forces in Eqs. (3), (4), (6), (7), and (8) if the two-parameter foundation is considered.

If a moving load has loaded lengths of d_1 and d_2 in the η- and y-directions and the load pressure (load per unit of area) q, the transformed load Q defined in Eqs. (5) and (9) can be obtained, respectively, by

$$Q(\xi,\zeta,\Omega) = 4\frac{\sin\frac{d_1\xi}{2}\sin\frac{d_2\zeta}{2}}{\xi\zeta}\mathrm{e}^{-\mathrm{i}\xi\eta_0}\mathrm{e}^{-\mathrm{i}\zeta y_0}\int_{-\infty}^{\infty} g(t)\mathrm{e}^{-\mathrm{i}\Omega t}\,\mathrm{d}t \tag{11}$$

$$Q(\xi,\zeta,t) = 4q\frac{\sin\frac{d_1\xi}{2}\sin\frac{d_2\zeta}{2}}{\xi\zeta}\mathrm{e}^{\mathrm{i}\overline{\Omega}t}\mathrm{e}^{-\mathrm{i}\xi\eta_0}\mathrm{e}^{-\mathrm{i}\zeta y_0} \tag{12}$$

where η_0 and y_0 are the coordinates of the center of the load and $g(t)$ is the variation in load amplitude with time.

To obtain the response to multiple loads, the superposition method can be used; however, care should be taken when the steady-state response due to the moving harmonic loads is to be obtained since the amplitude of the response at each point does not occur at the same time (Kim and Roesset, 1998; Kim and McCullough, 2003). An alternative to obtain the response to moving multiple loads is to use the combined transformed load calculated from the multiple loads (Kim et al., 2001, 2002).

3. Behavior under in-plane compressive forces and a moving load of constant amplitude

The dynamic displacement response and buckling of a plate on an elastic foundation subjected to in-plane compressive forces and a moving load of constant amplitude are investigated first. The material properties and geometry used in this study are listed in Table 1. The properties in the table have been selected arbitrarily within the practical ranges of the parameters for the pavement systems. The values of the parameters not shown in the table, such as load velocity, in-plane compressive force, and damping value, are considered within wide ranges.

3.1. Deflected shapes

If a plate is sufficiently large and the velocity is constant, the deflected shapes under the moving load of constant amplitude are the same at any instant along the moving axis. This means that the deflected shape is moving with the load.

Table 1
Properties of plate on elastic foundation and moving load

Properties of plate on elastic foundation	
E	1516 MPa
v	0.35
k	95 MN/m^3
h	152.4 mm
m	366 kg/m^2
Properties of moving load	
Total load	−40 kN
d_1 and d_2	152.4 mm

The effects of the in-plane compressive forces on the deflected shapes along the x- and y-axes are shown in Fig. 2, when a load velocity is 127 m/s (which is smaller than the critical velocity) and there is no damping. The 0 distance in the figure represents the location of the center of the load. The deflected shapes are symmetric with respect to the two axes that pass through the center of the moving load. For the deflected shapes along x-axis (or in the moving direction), as shown in Fig. 2a, there are clear upward peaks at the front and rear of the load, but those peaks are not pronounced in the deflected shapes along y-axis, as shown in Fig. 2b. If the in-plane compressive force exists in the x-direction, the maximum deflection and the peaks at the front and rear of the load become larger, and it makes the deflected shape along y-axis wider. In presence of the in-plane compressive force in the y-direction (P_y), the maximum deflection becomes larger and the deflected shape along y-axis narrower, but it does not seem to affect the peaks at the front and rear of the moving load. As can be seen in Eqs. (4), (6), (7), and (8), the presence of the in-plane compression reduces the xxx of the transfer function and makes the deflections larger.

The deflected shapes when there is damping in the system are shown in Fig. 3 for a load velocity of 127 m/s and a damping ratio of 10%. The deflected shapes along y-axis are symmetric with respect to the center of the load (Fig. 3b), but the deflected shapes in the moving direction are no longer symmetric as shown in Fig. 3a. The peak that occurs at the front of the load increases and the peak that occurs behind the load decreases. The maximum deflection does not occur exactly at the center of the load and it occurs slightly

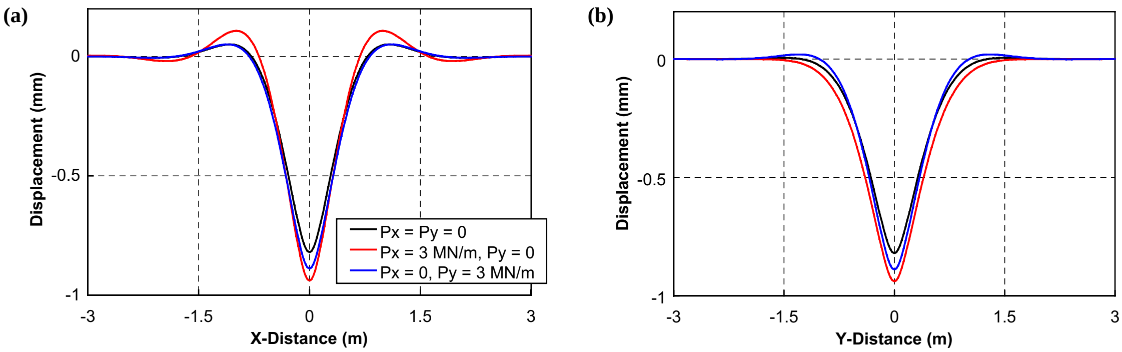

Fig. 2. Deflected shapes along: (a) x-axis (b) y-axis ($D = 0\%$, $V = 127$ m/s).

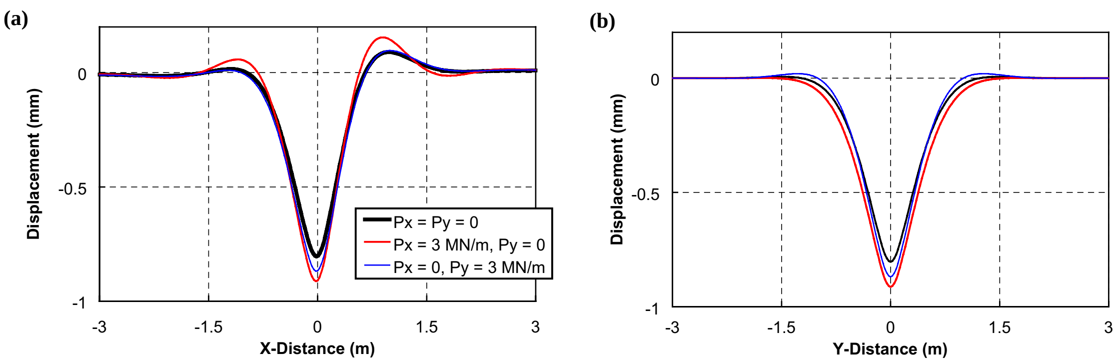

Fig. 3. Deflected shapes along: (a) x-axis (b) y-axis ($D = 10\%$, $V = 127$ m/s).

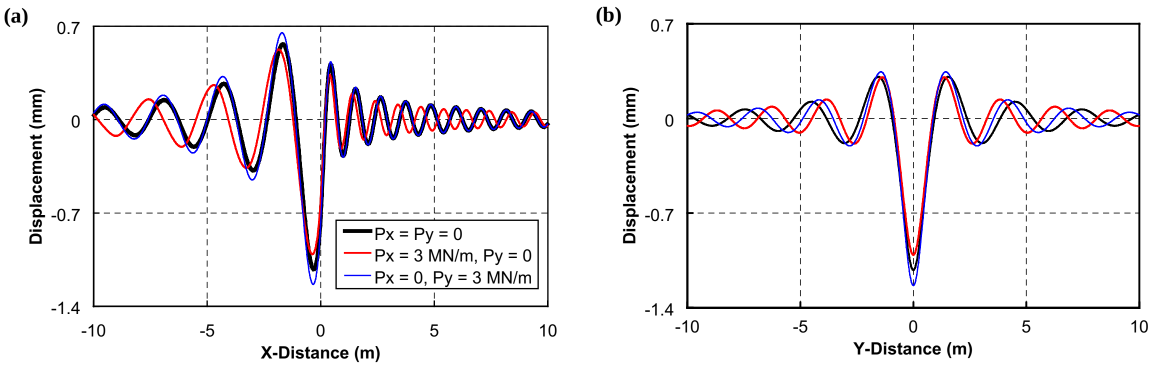

Fig. 4. Deflected shapes along: (a) x-axis (b) y-axis ($D = 5\%$, $V = 230$ m/s).

behind the center of the load. The lag between the center of the load and the position where the maximum deflection occurs is called distance lag (Kim and Roesset, 1998). The effects of the in-plane compressive forces on the deflected shapes when there is damping in the system are the same as those when there is no damping.

The deflected shapes when a velocity is 230 m/s and a damping ratio is 5% are shown in Fig. 4. As will be mentioned later, the velocity considered here (230 m/s) is higher than the critical (resonance) velocity. The deflected region is widely spread in this case. For the deflected shapes in the moving direction (Fig. 4a), the deflected shapes behind and ahead of the load are clearly different. The frequency of the displacement fluctuations in front of the load is larger than that behind the load. In presence of the in-plane compressive force in the x-direction, the frequency of the fluctuations in front of the load increases, but that behind the load decreases. The in-plane compression in the y-direction does not affect the fluctuation frequency. Fig. 4b shows the deflected shapes along y-axis at the x coordinate where the maximum deflection occurs. The frequency of the displacement fluctuations in the y-direction becomes larger with the in-plane compression either in the x-direction or in the y-direction.

3.2. Critical velocity and buckling force

The effect of the in-plane compression on the relationship between the maximum deflection and the load velocity is shown in Fig. 5. As the load velocity increases, the maximum deflection increases until the

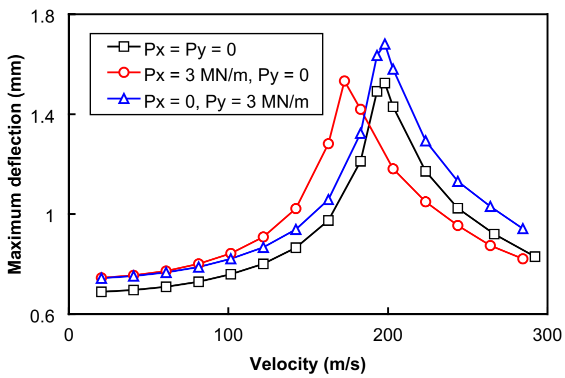

Fig. 5. Relationship between maximum deflection and velocity ($D = 5\%$).

velocity becomes close to the critical velocity and then decreases again. In presence of the in-plane compression in the moving direction, the critical velocity decreases. However, the in-plane compression in the y-direction does not affect the critical velocity. It just makes the maximum deflection larger. For velocities smaller than the critical velocities, the system having the in-plane compression in the moving direction has the largest maximum deflection compared with the systems having the in-plane compression in the y-direction with the same load magnitude and having no in-plane compression; however, it has the smallest maximum deflection for velocities larger than the critical velocities.

Fig. 6 shows the relationship between the distance lag and the velocity. The distance lag increases as the velocity increases. For velocities near and above the critical velocity, the distance lag becomes significantly larger. When there is the in-plane compression in the moving direction, the distance lag at a given velocity becomes slightly larger than that of the system having no in-plane compression for velocities smaller than the critical velocity and becomes significantly larger for velocities larger than the critical velocity. However, the in-plane compression in the y-direction affects the distance lag only for velocities much larger than the critical velocity.

To find an expression for the critical velocity and the critical in-plane compressive force (buckling force), a number of parametric studies were performed considering wide ranges of variables. It was found that the size of the loaded area, within the practical range of the tire print area, did not affect the critical velocity. It was also found that the critical velocity tended to increase very slightly with an increase in the damping

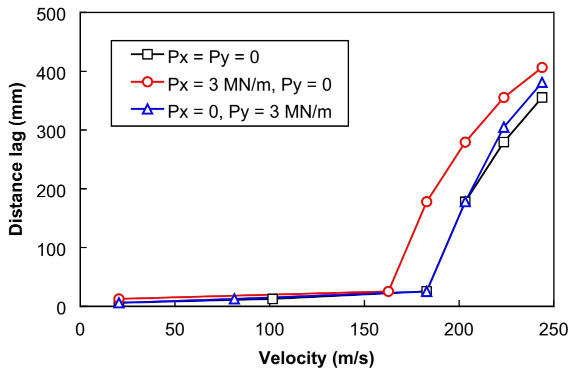

Fig. 6. Relationship between distance lag and velocity ($D = 5\%$).

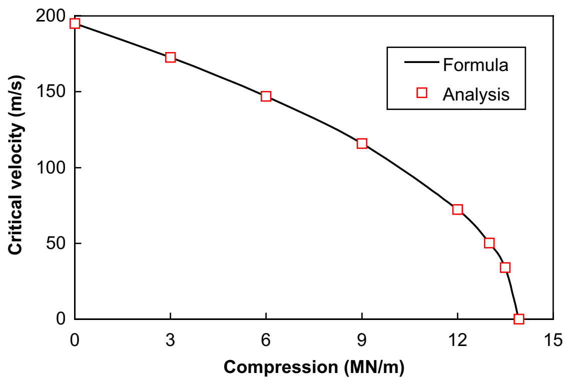

Fig. 7. Relationship between critical velocity and in-plane compression.

ratio, but the increase was small enough to be negligible. Finally, the critical velocity V_{cr0} for a given in-plane compression in the moving direction can be predicted by

$$V_{cr0} = \sqrt{\frac{2\sqrt{D_P k} - P_x}{m}} \quad (\text{when } P_x \leqslant 2\sqrt{D_P k}) \tag{13}$$

The critical in-plane compressive force in the moving direction for a given velocity can be obtained by solving Eq. (13) for P_x. Then,

$$P_{x\,cr} = 2\sqrt{D_P k} - mV^2 \tag{14}$$

As can be predicted with Eq. (14), the static buckling force (when $V = 0$) is $2\sqrt{D_P k}$, and the magnitude of the buckling force decreases as the velocity increases until the critical velocity. As mentioned previously, the in-plane compression in the y-direction does not affect the critical velocity. Accordingly, the buckling force in the y-direction is independent of the velocity and the same as the static buckling force. Fig. 7 shows the relationship between the critical velocity and the in-plane compression, obtained from the analysis and predicted by Eq. (13). The results from the analysis and the formula are identical.

4. Behavior under in-plane compressive forces and a moving harmonic load

The dynamic displacement response and buckling of a plate on an elastic foundation are investigated when subjected to in-plane compressive forces and a moving load with harmonic amplitude variation. First, the stability and the vibration of the system for a stationary harmonic load ($V = 0$) are studied. Then, the maximum displacements under a moving harmonic load are examined for various values of the velocity, load frequency, and in-plane compressive force. The expressions for the critical values of the velocity, frequency, and in-plane compressive force are finally developed. Since the in-plane compressive force in the y-direction does not affect the critical values of the velocity and frequency, only the in-plane compressive force in the moving direction is considered.

4.1. Response to a stationary harmonic load

The relationship between the maximum displacement and the load frequency for a stationary harmonic load ($V = 0$) is shown in Fig. 8. When there is no in-plane compression, the maximum displacement increases until the load frequency reaches the critical (resonance) frequency and then decreases. When there is the in-plane compression, two critical frequencies are observed. The first critical frequency is smaller than the second critical frequency, and it is the same as the critical frequency of the system having no in-plane compression. For a given frequency smaller than the first critical frequency of the system having the in-plane compression, the in-plane compression makes the maximum displacement larger; however, for a given frequency larger than the second critical frequency, the in-plane compression reduces the maximum displacement but the reduction becomes smaller and can be negligible as the load frequency increases.

It was also found that the flexural rigidity of the plate, size of the loaded area, and damping ratio did not affect the critical frequency of the stationary harmonic load when there is no in-plane compression. If the in-plane compression exists, however, the critical frequency is affected by many parameters but independent of the loaded area and damping ratio. The critical frequency in Hz (cps) of the stationary harmonic load, f_{cr0}, can be obtained by

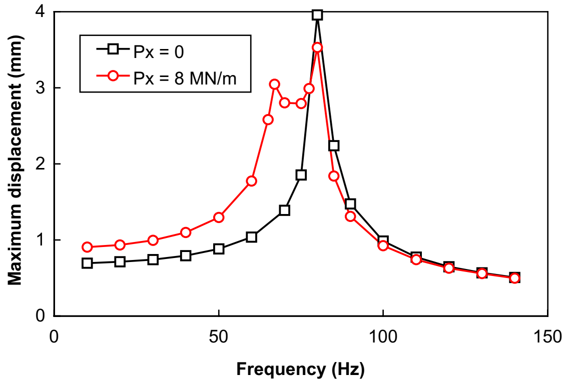

Fig. 8. Relationship between maximum displacement and load frequency for stationary harmonic load ($D = 1\%$).

$$f_{\mathrm{cr0}} = \frac{1}{2\pi}\sqrt{\frac{k}{m} - \frac{P_x^2}{4mD_\mathrm{P}}} \quad (\text{when } P_x \leqslant 2\sqrt{D_\mathrm{P}k}) \tag{15}$$

The second critical frequency is the same as the critical frequency of the system having no in-plane compression and can be determined by

$$f_{\mathrm{cr0\,2nd}} = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \tag{16}$$

The critical in-plane compressive force for a stationary harmonic load can be obtained by

$$P_{x\,\mathrm{cr}} = 2\sqrt{D_\mathrm{P}(k - 4m\pi^2\bar{f}^2)} \quad \left(\text{when } \bar{f} \leqslant \frac{1}{2\pi}\sqrt{\frac{k}{m}}\right) \tag{17}$$

where $\bar{f}$ is the frequency of the harmonic load in Hz. Fig. 9 shows the relationship between the critical frequency and the in-plane compression for a stationary harmonic load, obtained from the analysis and Eqs. (15) and (16). As the magnitude of the in-plane compressive force increases, the first critical frequency

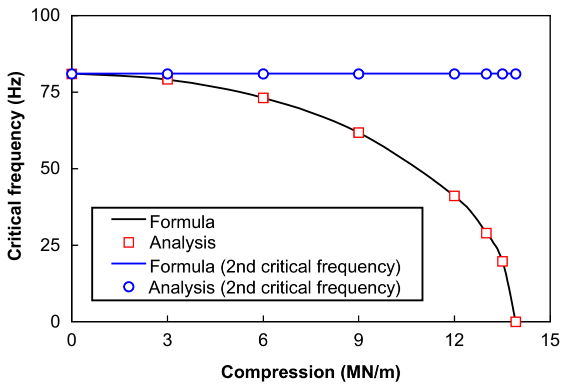

Fig. 9. Relationship between critical frequency of stationary harmonic load and in-plane compression.

becomes smaller but the second critical frequency is constant. The results from the analysis and the formulas are identical.

4.2. Response to a moving harmonic load

The relationship between the maximum displacement and the load frequency of the moving harmonic load is shown in Fig. 10a for a load velocity of 125 m/s that is smaller than the critical velocity of the moving load of constant amplitude (in this case, about 173 and 195 m/s for the systems with the in-plane compressive forces of 3 N/m and 0, respectively). The critical frequencies are smaller than those of the stationary harmonic load (compare with Fig. 8), and the critical frequency decreases with the in-plane compression. For a given frequency except for the frequencies near the critical frequencies, the maximum displacement becomes larger with the in-plane compression.

As shown in Fig. 10b, for a load velocity of 250 m/s that is larger than the critical velocities of the moving load of constant amplitude, the critical frequency increases with the in-plane compression. For low frequencies, the maximum displacement decreases with the in-plane compression. For frequencies larger than the critical frequencies, the increase in the maximum displacement caused by the in-plane compression, for a given frequency, becomes smaller as the frequency increases.

The relationship between the maximum displacement and the load velocity when a load frequency is 50 Hz that is smaller than the critical frequencies of the stationary harmonic load is shown in Fig. 11a. There are two critical velocities and both the first and second critical velocities decrease in presence of the in-plane compression. When a load frequency is 100 Hz that is larger than the critical frequencies of the stationary harmonic load, as shown in Fig. 11b, the peaks corresponding to the second critical velocities are very clearly observed but the peaks corresponding to the first critical velocities are not apparent. The in-plane compression makes the second critical velocity smaller.

4.3. Critical velocity, critical frequency, and critical in-plane compressive force

The relationship between the critical velocity and the load frequency is shown in Fig. 12. The critical velocity for a load frequency of 0 represents the critical velocity of the moving load of constant amplitude. As the load frequency increases, the first critical velocity decreases and the second critical velocity increases. For a given frequency, both the first and second critical velocities decrease with the existence of the in-plane compression. From a number of parametric studies, formulas that can be suggested to predict the first and

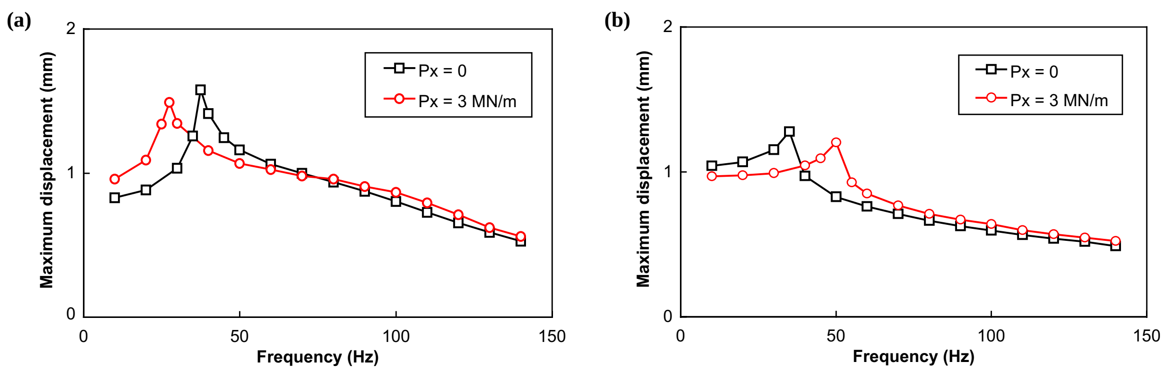

Fig. 10. Relationship between maximum displacement and load frequency ($D = 1\%$): (a) $V = 125$ m/s (b) $V = 250$ m/s.

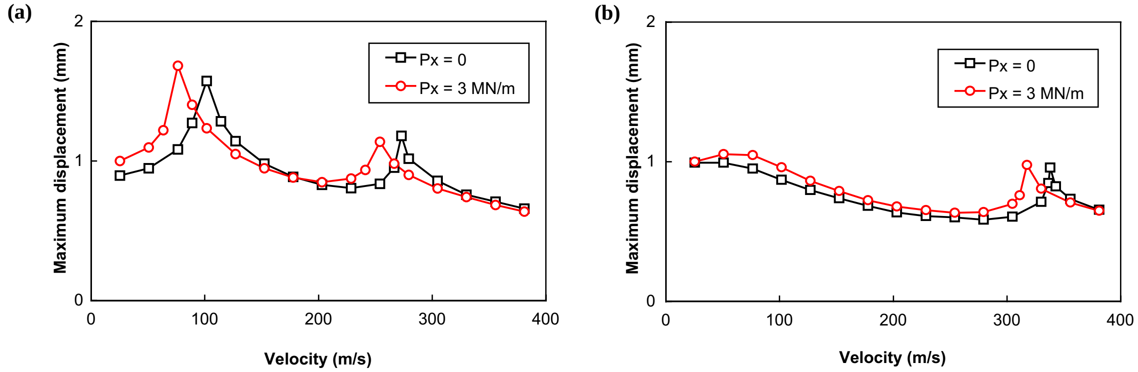

Fig. 11. Relationship between maximum displacement and velocity ($D = 1\%$): (a) frequency = 50 Hz, (b) frequency = 100 Hz.

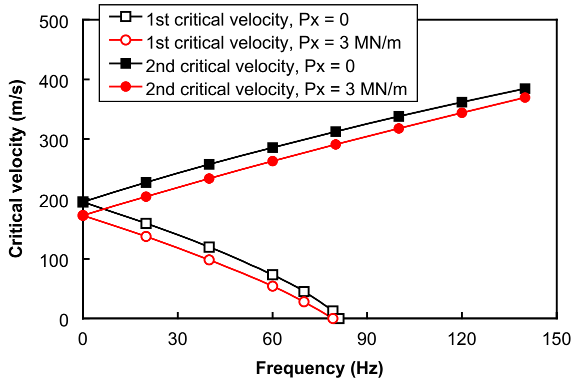

Fig. 12. Relationship between critical velocity and load frequency.

second critical velocities, V_{cr1} and V_{cr2} respectively, within the practical ranges of the variables for the pavement systems, were found for the moving harmonic load as follows:

$$V_{cr1} = V_{cr0}\left(1 - \frac{\bar{f}}{f_{cr0}}\right)^{0.72\alpha} \quad (\text{when } \bar{f} \leqslant f_{cr0}) \tag{18}$$

$$V_{cr2} = V_{cr0}\left(1 + \frac{\bar{f}}{f_{cr0}}\right)^{0.69\alpha} \tag{19}$$

with

$$\alpha = \frac{\sqrt{\frac{2\sqrt{D_P k}}{m}}}{V_{cr0}}\sqrt{1 - \frac{P_x^2}{4D_P k}} \quad (\text{when } P_x \leqslant 2\sqrt{D_P k}) \tag{20}$$

where V_{cr0} is the critical velocity of the moving load of constant amplitude defined by Eq. (13), $\bar{f}$ is the load frequency of the moving harmonic load in Hz, and f_{cr0} is the critical frequency of the stationary harmonic

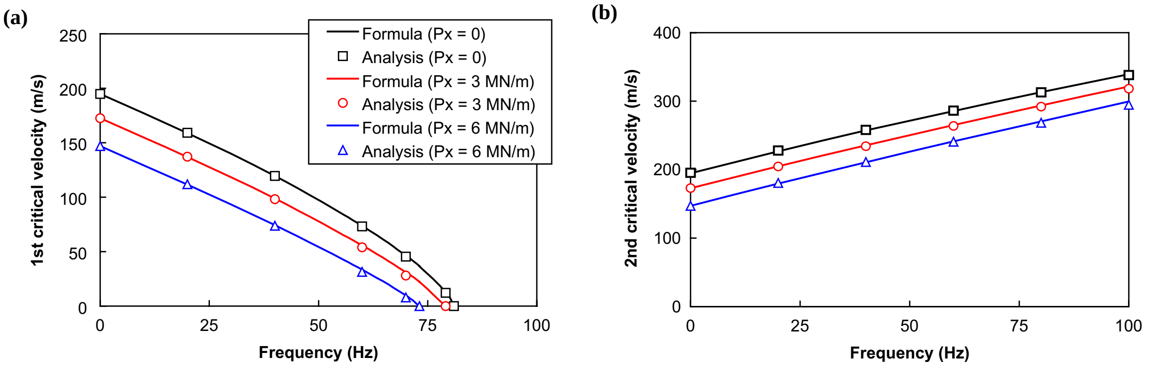

Fig. 13. Results from analysis and suggested formula for: (a) first and (b) second critical velocities.

load defined by Eq. (15). The critical frequency for a given velocity and a given in-plane compression can also be obtained solving Eqs. (18) and (19) for $\bar{f}$ and can be written as

$$f_{cr} = f_{cr0}\left\{1 - \left(\frac{V}{V_{cr0}}\right)^{\frac{1}{0.72\alpha}}\right\} \quad (\text{when } V \leqslant V_{cr0}) \tag{21}$$

$$f_{cr} = f_{cr0}\left\{\left(\frac{V}{V_{cr0}}\right)^{\frac{1}{0.69\alpha}} - 1\right\} \quad (\text{when } V \geqslant V_{cr0}) \tag{22}$$

Similarly, the critical in-plane compressive force for a given velocity and a given frequency can be determined solving Eqs. (21) and (22) for the in-plane compressive force P_x. The critical velocities obtained from the analysis and predicted by the suggested formulas are compared as shown in Fig. 13. The results from the suggested formulas and those of the analysis are in very good agreement.

5. Summary and conclusions

The dynamic displacement response of a plate on an elastic foundation has been investigated when the system is subjected to in-plane static compressive forces and a moving load with either constant or harmonic amplitude variations. For the steady-state response to a moving harmonic load and for the response to a moving load of constant amplitude, formulations were developed in the transformed field domains of time, space, and moving space, and solutions were obtained using FFT. Analyses were performed to investigate the effects of various parameters on the deflected shape, maximum displacement, and critical values of the velocity, frequency, and in-plane compression, and to examine how the in-plane compression affects the vibration and stability of the system. Expressions to predict the critical values of the velocity, frequency, and in-plane compression were developed. The analysis results point to the following conclusions.

- When the system is subjected to a moving load of constant amplitude:
  - For velocities smaller than the critical velocity, if the in-plane compressive force is applied in the moving direction (x-direction), the maximum deflection and the peaks at the front and rear of the load become larger and the deflected shape along y-axis becomes wider. In presence of the in-plane compressive force in the y-direction, the maximum deflection becomes larger and the deflected shape along

y-axis narrower, but it does affect the peaks at the front and rear of the moving load along x-axis.
  - For velocities larger than the critical velocity, the deflected region is widely spread and the deflected shapes behind and ahead of the load are clearly different. The frequency of the displacement fluctuations in front of the moving load is larger than that behind the load. In presence of the in-plane compressive force in the x-direction, the frequency of the displacement fluctuations in front of the moving load increases, but that behind the moving load decreases. The in-plane compression in the y-direction does not affect the displacement fluctuation frequency along x-axis. The frequency of the displacement fluctuations in the y-direction becomes larger with the in-plane compression either in the x-direction or in the y-direction.
  - The critical velocity decreases in presence of the in-plane compression in the moving direction. The in-plane compression in the y-direction does not affect the critical velocity. The in-plane buckling force in the moving direction decreases as the velocity increases.
  - The distance lag for a given velocity increases with the existence of the in-plane compression. The distance lag becomes significantly larger for velocities larger than the critical velocity.
- When the system is subjected to a stationary harmonic load:
  - In presence of the in-plane compression, two critical frequencies are observed. The first critical frequency is smaller than the second, and the second critical frequency is the same as the critical frequency of the system without the in-plane compression.
  - The critical frequency becomes smaller as the magnitude of the in-plane compressive force increases. In other words, the in-plane buckling force decreases as the load frequency increases.
- When the system is subjected to a moving harmonic load:
  - The in-plane compression in the y-direction does not affect the critical values of the velocity and frequency.
  - In presence of the in-plane compression in the x-direction, the critical frequency decreases when the load velocity is smaller than the critical velocity of the moving load of constant amplitude. However, for velocities larger than the critical velocity of the moving load of constant amplitude, the critical frequency increases with the existence of the in-plane compression in the x-direction.
  - For frequencies smaller than the critical frequency of the stationary harmonic load, two critical velocities exist and those critical velocities decrease when the in-plane compression in the x-direction is applied. For frequencies larger than the critical frequency of the stationary harmonic load, only the second critical velocity is clearly observed, and it decreases with the application of the in-plane compression.

References

Bay, J.A., Stokoe II, K.H., Jackson, J.D., 1995. Development and preliminary investigation of a Rolling Dynamic Deflectometer. Transportation Research Record 1473, 43–54.

Brunner, R.J., 1975. Prestressed pavement demonstration project. Transportation Research Record 535, 62–72.

Cable, N.D., McCullough, B.F., Burns, N.H., 1986. New concepts in prestressed concrete pavement. Transportation Research Record 1099, 1–12.

Filonenko-Borodich, M.M., 1940. Some approximate theories of the elastic foundation. Uchenyie Zapiski Moskovskogo Gosudarstuennogo Universiteta Mechanika 46, 3–18, (in Russian).

Foinquinos, R., Roesset, J.M., 1995. Dynamic nondestructive testing of pavements. Geotechnical Engineering Report GR95-4, Geotechnical Engineering Center, The University of Texas at Austin.

Huang, M.-H., Thambiratnam, D.P., 2002. Dynamic response of plates on elastic foundation to moving loads. ASCE Journal of Engineering Mechanics 128 (9), 1016–1022.

Huang, Y.H., 1993. Pavement analysis and design. Prentice Hall, New Jersey.

Kerr, A.D., 1964. Elastic and viscoelastic foundation models. Journal of Applied Mechanics 31 (3), 491–498.

Kerr, A.D., Dallis, W.A., 1985. Blowup of concrete pavements. ASCE Journal of Transportation Engineering 111 (1), 33–53.

Kim, S.-M., McCullough, B.F., 2003. Dynamic response of plate on viscous Winkler foundation to moving loads of varying amplitude. Engineering Structures 25 (9), 1179–1188.
Kim, S.-M., Roesset, J.M., 1998. Moving loads on a plate on elastic foundation. ASCE Journal of Engineering Mechanics 124 (9), 1010–1016.
Kim, S.-M., Roesset, J.M., Stokoe II, K.H., 1999. Numerical simulation of Rolling Dynamic Deflectometer tests. ASCE Journal of Transportation Engineering 125 (2), 85–92.
Kim, S.-M., Won, M.C., McCullough, B.F., 2001. Transformed field domain analysis of pavements subjected to moving dynamic tandem-axle loads and integrating their effects into the CRCP-10 program. Research Report 1831-5, Center for Transportation Research, The University of Texas at Austin.
Kim, S.-M., Won, M.C., McCullough, B.F., 2002. Dynamic stress response of concrete pavements to moving tandem-axle loads. Transportation Research Record—Journal of the Transportation Research Board 1809, 32–41.
Liu, C., McCullough, B.F., Oey, H.S., 2000. Response of rigid pavements due to vehicle-road interaction. ASCE Journal of Transportation Engineering 126 (3), 237–242.
Nasim, M.A., Karamihas, S.M, Gillespie, T.D, Hansen, W., Cebon, D., 1991. Behavior of a rigid pavement under moving dynamic loads. Transportation Research Record 1307, 129–135.
Okamoto, P.A., Tayabji, S.D., 1995. Instrumentation and evaluation of prestressed pavement section in Pennsylvania. Transportation Research Record 1505, 103–111.
Pasternak, P.L., 1954. On a new method of analysis of an elastic foundation by means of two foundation constants (in Russian). Gosudarstvennoe Izdatelstvo Literaturi po Stroitelstvu i Arkhitekture, Moscow.
Powers, R.L., Zaniewski, J.P., 1987. Nine-year performance evaluation of Arizona's prestressed concrete pavement. Transportation Research Record 1136, 1–11.
Vlasov, V.Z., Leontev, U.N., 1966. Beams, plates and shells on elastic foundations (translated from Russian). Israel Program for Scientific Translation Jerusalem, Israel.
Westergaard, H.M., 1925. Stresses in concrete pavements computed by theoretical analysis. Public Roads 7, 25–35.